Density-Tuned Curve Regularization

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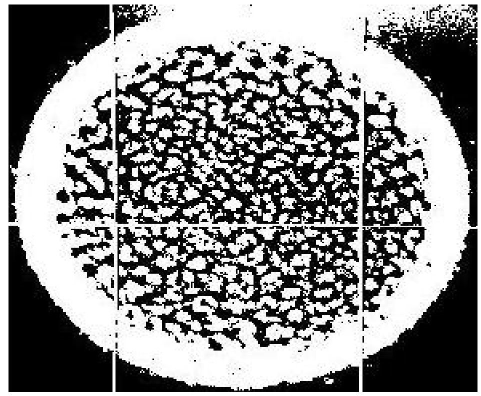
Introduction

Our aim is to analyze the behavior of closed curves extracted from segmented images. Our curves represent material boundaries in images of physical experiments. Before reaching us, those images undergo a lot of preprocessing, which distorts the original shape of the curves. That is why, prior to analyzing them, we have to regularize the curves in order to ensure the analysis is correct. For example, in the figure on the right we have a segmentation of an image of a hemisphere of fracturing metal. We want to study the boundaries of the metal pieces in order to learn more about where and how the fracturing occurs. Since the segmentation was achieved through simple thresholding of the images, there is a lot of noise around the boundaries, and we would like to find a way to correct this noise. Usually, we do not know how the curves look, since the behavior of materials is not always physically understood. Sometimes we are allowed, however, to make some assumptions about their general shapes. We would like to use those assumptions to regularize the curves in a way that they are a better representation of the physical phenomena going on in the experiment. To do that, we use curve densities which we describe in detail below.

Background

Our problem is an ill-posed inverse problem. We are trying to retrieve the original images, but we have lost some information in the process of acquiring the data. Regularization is an efficient way to solve such inverse problems. It uses some a priori information about the solution to stabilize the problems. Tikhonov was the first one to introduce a penalty term to the solving of the least-squares problem [1]. Later his model became a general scheme for solving inverse problems:



This is an image of a hemisphere of fracturing metal. The small white shapes represent the different pieces the metal breaks into. The boundaries of those pieces are not the true boundaries, since they get distorted during the segmentation.

$$\min_u F(u) = D(u,d) + \lambda R(u). \qquad (1)$$

We minimize $F(u)$ where u is the reconstructed image u. $D(u,d)$ is a metric showing the distance between the reconstructed image u, and the data we have, d. $R(u)$ is the regularization term in which we force the solution to have certain properties. λ is a parameter balancing the minimization of the two terms.

Density-Tuned Regularization

To solve our problem we use the general regularization scheme. We are trying to minimize the distance between the original and reconstructed images, and regularize the result using some prior knowledge about the density of the curve. We use the area density of the curve which corresponds to the two-dimensional Housdorff density. While in the theoretical literature one is primarily concerned with these densities in the limit as the scale approaches zero, we use a series of approximations to obtain estimates for the properties of the curve on various scales. To calculate density at some point we consider a circle centered at that point on the curve. The area density is the ratio of the area of the region interior both to the curve and the circle, and the area of the circle. We use the area density property of describing the behavior of the curve locally to tune our curves to behave in a way to fulfill our expectations. For example, if we want the curve to be locally flat, then its area density should be close to 1/2.

We can achieve our goal through density-tuned regularization of the curve. We use a 2D binary image, which we obtain by setting the interior of the curve to one, and the background to zero. To get an explicit area density representation, we convolve that image with a disk of some radius. The value of this convolution at the edges of the image is equal to the area density of the curve at that point. We use regularization to force this value to be close to the area density value we desire the curve to have. We define our metric and regularization terms as follows:

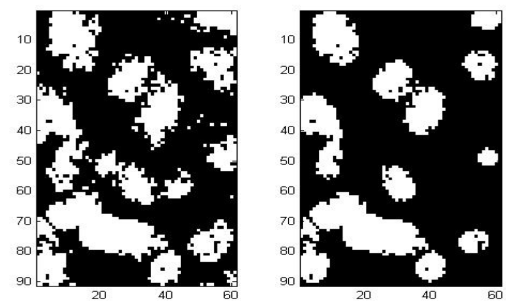
$$D(u,d) = \int (u-d)^2 dx \tag{2}$$

$$R(u) = \int (u * B - c)^2 |\nabla u| dx \tag{3}$$

Note that u and d are binary images. $B(x,r)$ is a disk of radius r centered at x . $(u*B-c)^2$ is our tuning term, where c is a constant corresponding to the desired area density, and $*$ is the convolution operator. $|\nabla u|$ forces the tuning to occur mainly on the edges of the image. We substitute (2) and (3) in (1), and we get a formula for $F(u)$.

Implementation

To minimize $F(u)$ we look for minima in the opposite direction of the gradient. However, if we apply the gradient descent method directly, we run into a problem, since we need to keep u binary. What we do is first to calculate the first derivative of $F(x)$. Then we update our binary image in the appropriate direction. We set the binary image to one where the derivative is positive, and to zero where the derivative is negative. The rest of the binary image remains unchanged. This way we update the image only on the edges. The following figure displays some of the results we achieved with our algorithm.



The left image represents a zoom in on a few pieces of metal from the hemisphere shown in the first figure. The right image shows regularization of the left image, in which we want the boundaries of the pieces to be locally flat ($c = 1/2$).

Future

The idea of using densities for curve regularization is still in an early stage of development. There is still potential for more work and results in that direction. We developed an analytical representation of the problem and implemented basic algorithms for the solution. Our algorithms can definitely be improved and sped up. We also believe that the specific properties of the densities can be used to make the tuning more dynamic. We would like to tune the curves to different density values at different locations, or do the regularization on different scales. The final aim is to apply our regularization scheme to images from actual LANL experiments.

Acknowledgements

Los Alamos Report LA-UR-05-6652. Funded by the Department of Energy under contracts W-7405-ENG-36 and the DOE Office of Science's Advanced Scientific Computing Research (ASCR) program in Applied Mathematical Sciences.

References

[1] Aubert G., Kornprobst P., Mathematical problems in image processing , New York: Springer, 2001.